

# A STOCHASTIC ENERGY MARKET MODEL FOR EVALUATING THE INTEGRATION OF WIND ENERGY

Heike BRAND<sup>+#</sup>, Christoph WEBER<sup>+</sup>, Peter MEIBOM<sup>°</sup>,  
Rüdiger BARTH<sup>+</sup>, Derk Jan SWIDER<sup>+</sup>

<sup>+</sup>Institute of Energy Economics and the Rational Use of Energy (IER), University of Stuttgart, Hessbruehlstr. 49a, 70565 Stuttgart, Germany  
<sup>°</sup>RISOE National Laboratory, Roskilde, Denmark

<sup>#</sup>Dipl.-Wirtsch-Ing. Heike Brand. Tel.: +49-711-780-6152; Fax: +49-711-780-3953. E-Mail: Heike.Brand@ier.uni-stuttgart.de.

## Abstract

*A fast growth of the installation of wind turbines has been experienced in several European countries. The introduction of substantial amounts of wind power in a liberalized electricity system will have serious impacts on the markets: market prices will change since the marginal production costs of wind power are very low, and to maintain the stability of the power system, larger amounts of frequency-responding spinning as well as supplemental power reserve will be needed. Moreover transmission bottlenecks may occur between various regions.*

*Modelling explicitly the stochastic behaviour of wind generation and taking into account the prediction error is crucial for an evaluation of the costs of the integration of wind power. In this paper, a stochastic linear programming model is described for the efficient power market operation using the principle of rolling planning on an hourly basis. The model includes four markets: i) a day-ahead market for physical delivery of electricity, ii) an intraday-market, iii) a day-ahead market for automatically activated reserve power and iv) a market for district heating and process heat. Time series for the wind power production rely on time series for wind speed data and for the prediction error for wind speed. Aggregation of wind power generation reflects the spatial distribution of the wind power stations in each region. Market restrictions, capacity restrictions, restrictions for down regulation, minimum operation and shut down times and hydro storages are included in this model.*

*The application of the model to the German market is shown using a three stage model where 3 regions and 40 different types of units are defined. The results indicate transmission bottlenecks caused by varying wind power production in Germany. Price variations between the day-ahead market price and the intraday market price occur whenever the forecast for wind speed shows significant differences to the real wind speed.*

Keywords: wind power, stochastic optimization, day-ahead market, regulatory market

## 1 Introduction

The process towards liberalized electricity markets has been going on for some years. The EU-directive on common rules for the internal market in electricity states that each member state has the right of access to the electricity transmission and distribution grids, thus opening the concept of free electricity trade in Europe. All EU countries are at least in the transition phase of liberalizing their electricity industry. Electricity exchange markets have developed to facilitate electricity trade and now exist in several countries, among these Germany, the Netherlands, England, Norway, Sweden, Finland and Denmark.

In recent years a number of European countries have simultaneously experienced a fast growth in the installation of wind turbines, e.g. Germany, Spain and Denmark. It is very probable that these fast growth rates of wind power will continue in the years to come, which is also reflected in the 40000 MW target for wind power in 2010 in the European Countries (European Commission, 2000).

The introduction of substantial amounts of wind power in a liberalized electricity system will impact both the technical operation of the electricity system and the electricity market. In order to cope

with the fluctuations in the wind power production, other units in the power system have to be operated more flexibly to maintain the stability of the power system. Technically this means that larger amounts of wind power will require increased capacities of spinning and non-spinning power reserves and an increased use of these reserves. This implies generally a more frequent operation of the power plants in less efficient part-load operation. Moreover, if wind power is concentrated in certain (mostly coastal) regions, increased wind power generation may lead to bottlenecks in the transmission and distribution networks.

Economically, these changes in system operation have certainly cost and consequently price implications. Moreover they may also impact the functioning and the efficiency of certain market designs. Even if the wind power production is not bidded into the spot market, the feed-in of the wind power will affect the spot market prices, since it influences the balance of demand and supply.

As substantial amounts of wind power will require increased reserves, the prices on the regulating power markets are furthermore expected to increase. Yet this is not primarily due to the fluctuations of wind power itself but rather to the (partial) unpredictability of wind power. If wind power were fluctuating but perfectly predictable, the conventional power plants would have to operate also in a more variable way, but this operation could be scheduled on a day-ahead basis and settled on conventional day-ahead spot markets. It is the unpredictability of wind power which requires an increased use of reserves with corresponding price implications.

In order to analyse adequately the market impacts and the integration costs of wind power it is therefore essential to model explicitly the stochastic behaviour of wind generation and to take the forecast errors into account. In an ideal, efficient market setting, all power plant operators will take into account the prediction uncertainty when deciding on the unit commitment and dispatch. This will lead to changes in the power plant operation compared to an operation scheduling based on deterministic expectations, since the cost functions for power production are usually non-linear and not separable in time. E.g. even without fluctuating wind power, start-up costs and reduced part-load efficiency lead to a trade-off for power plant operation in low demand situations, i.e. notably during the night. Either the power plant operator chooses to shut down some power plants during the night to save fuel costs while operating the remaining plants at full output and hence optimal efficiency. Or he operates a larger number of power plants at part load in order to avoid start-up costs in the next morning. This trade-off is modified if the next increase in demand is not known with (almost) certainty as with early morning load increases but is stochastically distributed. So in an ideal world, where information is gathered and processed at no cost, power plant operators will anticipate possible future wind developments and adjust their power plant operation accordingly.

The model presented in the following describes such an ideal and efficient market operation by using a stochastic linear programming model, which depicts 'real world optimization' on the power market on an hourly basis with rolling planning. The commitment and dispatch of the generating units in the electricity system are governed by two main markets: (i) a day-ahead market for physical delivery of electricity and (ii) an intraday market or balancing power market, where the transmission system operators buy balancing power offered by flexible generating units and flexible electricity consumers in the system. With efficient markets, i.e. also without market power, the market results will correspond to the outcomes of a system-wide optimization as described in the following. The cost and price effects derived for the integration of wind energy in this model should then provide a lower bound to the magnitude of these effects in the real, imperfect world.

This paper is organized as follows: Section 2 presents a stochastic linear multi-stage model to study the effect of wind power on market prices in the different European markets (day-ahead, and intraday). Section 3 discusses the methodology to create scenarios for the wind power production, which are needed as input for the stochastic model. Section 4 illustrates the applicability of the model to the German market. Section 5 provides some conclusions.

## 2 Model

The model analyses power markets based on a description of generation, transmission and demand, combining the technical and economical aspects and it derives electricity market prices from marginal system operation costs. The model includes markets for three types of products: day-ahead market power, district and process heat, and regulating power. The model optimizes the unit commit-

ment and dispatch taking into account the trading activities of the different actors on the different energy markets. Additionally different restrictions such as transmission constraints or capacity constraints of the power and heat generating units are taken into account. An approximation for modelling minimum operation times and minimum shut down times in a linear way is included into the model definition. The proposed market model is defined as a stochastic linear programming model. The stochastic part is presented by a scenario tree for possible wind power generation for the different hours.

In stochastic multi-stage linear recourse models, there exist two types of decisions: decisions that have to be taken immediately and decisions that can be postponed. The first kind of decisions are called “root” decisions, as they have to be decided “here and now” and before the uncertain future is known. The second kind of decisions is called “recourse decisions”. They are taken after some of the uncertain parameters are known. With these “recourse decisions” actions can be started which might possibly revise the first decisions. In the case of a power system with wind power, the power generators have to decide on the amount of electricity they want to sell at the spot market before the precise wind power production is known (root decision). In most European countries this decision has to be taken at least 12-36 hours before the delivery period. And as the wind power prediction is not very accurate, recourse actions are necessary in most cases when the delivery period is in the near future (recourse decisions).

In a liberalized market environment it is often possible not only to change the unit commitment and dispatch, but even to trade electricity at the hour-ahead market. In this extended model three electricity markets and one market for heat are included in the planning model:

1. A day-ahead market for physical delivery of electricity where the EEX market at Leipzig, Germany is taken as the starting point. In the following, this market is called the day-ahead market.

2. An intra-day market for handling deviations between expected production and consumption agreed upon on the day-ahead market and the realized values of production and consumption in the actual operation hour. Regulating power can be traded up to one hour before delivery at the intra-day market. Both flexible producers and flexible consumers offer regulating power at this market. In our model the demand for regulating power is caused by the forecast errors connected to the wind power production.

3. A day-ahead market for automatically activated reserve power (frequency activated or load-flow activated). The demand for these ancillary services is determined exogenously to the model. This market will be called the ancillary services market.

4. Due to the interactions of CHP plants with the day-ahead and intra-day market, a market for district heating and process heat is included in model.

## 2.1 Parameters and decision variables:

### Sets:

$t, T$	: Index /set of time steps
$T^{OPTIM\_PERIOD}$	: Number of time steps
$s, S$	: Index/ set of scenarios
$i, I$	: Index/ set of units
$r, R$	: Index/ set of regions
$R_r^{NEIGHBOUR}$	: Set of regions, which are the neighbour regions of region r
$a, A$	: Index/ set of areas
$k$	: Index set of price-elastic intervals
$K^{UP}, K^{DOWN}$	: Set of flexible price intervals for increasing/decreasing the nominal demand
$I^{ELEC}, I_r^{ELEC}$	: Set of power producing units, set of power producing units in region r
$I^{ELEC\_ONLY}$	: Set of units producing only power

$I^{CHP}$	: Set of combined heat and power producing units
$I^{HEAT}, I_a^{HEAT}$	: Set of heat producing units, set of heat producing units in area a
$I^{EXTRACTION}$	: Set of units with extraction-condensing turbines
$I^{BACKPRESSURE}$	: Set of units with backpressure turbines
$I^{HYDRO}, I_r^{HYDRO}$	: Set of hydro storages, set of hydro storages in region r

**Parameters:**

$d_{r,t}^{ELEC}$	: Nominal load demand forecast for time step t region r
$d_{a,t}^{HEAT}$	: Heat demand forecast for time step t in area a
$d_{r,t}^{ANC,UP}, d_{r,t}^{ANC,DOWN}$	: Demand for spinning reserve (up-regulation/down regulation)
$P_{r,t}^{EXPECTED\_WIND}$	: Expected wind power production in region r at time step t
$P_{r,s,t}^{ACTUAL\_WIND}$	: actual wind power production capacity in region r, in scenario s at time step t
$P_{r,k,t}^{FLEXIBLE,DAY-AHEAD}$	: holds the price levels of individual steps in the electricity demand function
$c_i^{OPERATION}$	: cost function of unit i for operation costs
$s_i$	: cost function of unit i for start-up costs
$p_i^{MAX\_PROD}$	: Maximum output of power of unit i
$p_i^{MIN\_PROD}$	: Minimum output of unit i, when unit is online
$q_i^{MAX\_PROD}$	: Maximum output of heat of unit i
$t_i^{MIN\_OP}$	: Minimum operation time of unit i
$t_i^{MIN\_SD}$	: Minimum shut down times of unit i
$\delta_i^{CB}$	: Heat ratio of CHP turbine i
$\gamma_i$	: Reduction of electric power production due to heat production of CHP turbine i
$w_i^{MAX}$	: maximum capacity of the pump of hydro-storage i
$v_i^{MAX}$	: maximum capability of hydro-storage (reservoir) i
$P_{r,t}^{WATERVALUE}$	: value of water in hydro-storages in region r at time step t
$i_{i,t}^{INFLOW}$	: inflow into hydro-reservoir i at time step t

**Decision variables:**

$P_{i,s,t}, Q_{i,s,t}$	: Power / Heat Output of unit i in scenario s, at time step t
$P_{i,t}^{DAY\_AHEAD}$	: Power of turbine i sold to day-ahead market, at time step t
$P_{i,s,t}^+, P_{i,s,t}^-$	: Down / up-regulation for balancing market of turbine i in scenario s
$P_{i,t}^{ANC,+}, P_{i,t}^{ANC,-}$	: Contribution of unit i to spinning reserve (down/up-regulation)
$P_{i,s,t}^{ONLINE}$	: Online Capacity of unit i at time step t
$P_{r,\bar{r},s,t}$	: Transmission of power from region r to region $\bar{r}$ in scenario s
$P_{r,\bar{r}}^{TRANS,DAY-AHEAD}$	: Planned transmission when bidding on the day-ahead market
$P_{r,\bar{r},s,t}^{TRANS,+}, P_{r,\bar{r},s,t}^{TRANS,-}$	: Contribution to up-regulation/down-regulation at balancing market in region $\bar{r}$ by increased/decreased transmission of power from region r to region $\bar{r}$ in scenario s

$W_{i,s,t}$	: Pumping capacity of hydro-storage i in scenario s
$W_{i,t}^{DAY-AHEAD}$	: Fixed pumping capacity of hydro-storage i
$W_{i,s,t}^+, W_{i,s,t}^-$	: Down/up-regulation for balancing market of pump of hydro-storage i in scenario s
$W_{i,t}^{ANC,+}, W_{i,t}^{ANC,-}$	: Contribution of pump of hydro-storage i to spinning reserve (down/up-regulation)
$V_{i,s,t}$	: Content of Hydro-Storage i in scenario s
$D_{r,k,t}^{FLEX\_DAY-AHEAD}$	: Amount of increased/decrease demand (price-elastic) in region r

## 2.2 Objective Function and Restrictions

The model is formulated as a general stochastic unit commitment model. The technical consequences of the consideration of the stochastic behaviour of the wind power generation is the partitioning of the decision variables for power output as well for the electricity consumption and for the transmissions power: one part describes the different quantities at the day-ahead market (thus they are fixed and do not vary for different scenarios). The other part describes contributions at the intraday-market both for up- and down-regulation. The latter consequently depends on the scenarios. So for the power output of the units i at time t in scenario s we find  $P_{i,s,t} = P_{i,t}^{DAY-AHEAD} + P_{i,s,t}^+ - P_{i,s,t}^-$ . The variable  $P_{i,t}^{FIXED}$  denotes the energy sold at the day-ahead market and has to be fixed the day before. Therefore it does not vary for different scenarios.  $P_{i,s,t}^+$  and  $P_{i,s,t}^-$  denote the positive and negative contributions to the regulating power. Analogously the decision variables for the electricity consumption and for the transmissions power are defined accordingly.

The model is defined as a multi-regional model: each country is sub-divided into different regions, and the regions are sub-divided into different areas.

The objective function (1) tries to minimize the costs in the whole system, which corresponds to the maximization of producers' and consumers' surplus.

$$\min \left\{ \begin{aligned} & \sum_{s=1}^S \pi_s \sum_{t=1}^T \sum_{i \in I^{ELEC\_ONLY}} (c_i^{OPERATION}(P_{i,s,t}, P_{i,s,t}^{ONLINE}) + s_i(P_{i,s,t}^{ONLINE}, P_{i,s,t-1}^{ONLINE})) \\ & + \sum_{s=1}^S \pi_s \sum_{t=1}^T \sum_{i \in I^{CHP}} (c_i^{OPERATION}(P_{i,s,t}, Q_{i,s,t}, P_{i,s,t}^{ONLINE}) + s_i(P_{i,s,t}^{ONLINE}, P_{i,s,t-1}^{ONLINE})) \\ & + \sum_{s=1}^S \pi_s \sum_{t=1}^T \sum_{i \in I^{HEAT\_ONLY}} c_i^{OPERATION}(Q_{i,s,t}) \\ & - \sum_{s=1}^S \pi_s \sum_{t=1}^T \sum_{i \in I^{HYDRO}} V_{i,s,t} P_{i,t}^{WATERVALUE} \\ & + \sum_{t=1}^T \sum_{r \in R} \sum_{k \in K^{UP}} D_{r,k,t}^{FLEX\_DAY-AHEAD} P_{r,k,t}^{FLEXIBLE\_PRICE} \\ & - \sum_{t=1}^T \sum_{r \in R} \sum_{k \in K^{DOWN}} D_{r,k,t}^{FLEX\_DAY-AHEAD} P_{r,k,t}^{FLEXIBLE\_PRICE} \end{aligned} \right\} \quad (1)$$

The first two sums in (1) describe the operation and start-up costs of condensing turbines, of CHP turbines like backpressure turbines, gas turbines and extraction-condensing turbines. The third sum models the operation costs of heat boilers. The hydro-reservoirs are included into the model and

their power production costs are models through water values, which are calculated with the help of a long-term model developed by Hans Ravn (Dueholm, Ravn 2004). The model includes price-elastic consumer demand, which is done by defining a step-wise demand function. The change of the customers' utility relative to the change of their demand is represented through the last two sums in (1).

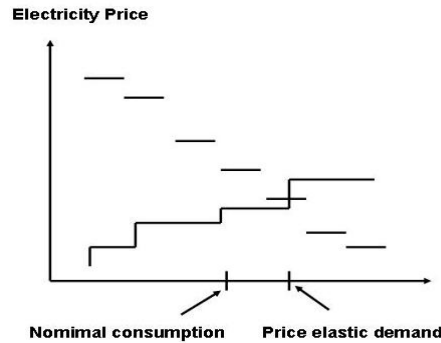
### A. Market restrictions for the balance of supply and demand

The demand constraint is split up into two constraints: one balance equation for the power sold at the day-ahead market and one balance equation for the power sold at the intraday market. The constraint for the time steps, where the day-ahead market is not fixed yet, is defined in (2).

$$\begin{aligned}
 & \sum_{i \in I_r^{ELEC}} P_{i,s,t} - \sum_{i \in I_r^{HYDRO}} W_{i,s,t} + \sum_{\bar{r} \in R_r^{NEIGHBOUR}} (P_{r,r,t}^{TRANS,+} - P_{r,r,t}^{TRANS,-}) + p_r^{EXPECTED\_WIND} \\
 & = d_{r,t}^{ELEC} + \sum_{k \in K^{UP}} D_{r,k,t}^{FLEX\_DAY-AHEAD} - \sum_{k \in K^{DOWN}} D_{r,k,t}^{FLEX\_DAY-AHEAD} \\
 & \forall t \in T^{NOT\_FIXED}, \forall r \in R
 \end{aligned} \tag{2}$$

The equation requires that all the power produced by the units in one region minus the power the hydro storages need for the pumping plus the import-export balance plus the expected wind power production has to be equal to the price-elastic demand in that region. The price-elastic demand consists of two parts: the nominal value at time t in region r  $d_{r,t}^{ELEC}$  and an elasticity function which specifies the relationship between quantity and the price for the deviation from the nominal profile. An illustration can be found in Figure 1.

**Figure 1: Marginal-Costs Offer Function and Price-Flexible Demand Function for Electricity**



The balance equation for the balancing market is described by the following equation:

$$\begin{aligned}
 & \sum_{i \in I_r^{ELEC}} (P_{i,s,t}^+ - P_{i,s,t}^-) + \sum_{i \in I_r^{HYDRO}} (W_{i,s,t}^+ - W_{i,s,t}^-) + \sum_{\bar{r} \in R_r^{NEIGHBOUR}} (P_{r,r,t}^{TRANS,+} - P_{r,r,t}^{TRANS,-}) \\
 & + \sum_{\bar{r} \in R_r^{NEIGHBOUR}} (P_{r,r,t}^{TRANS,-} - P_{r,r,t}^{TRANS,+}) - P_{r,s,t}^{WIND,-} = P_{r,t}^{WIND\_EXPECTED} - P_{r,s,t}^{WIND\_ACTUAL} \\
 & \forall r \in R, \forall s \in S, \forall t \in T
 \end{aligned} \tag{3}$$

Equation (3) says that the up and down regulation of the different turbines and the up and down regulation of the different pumps as well as the up and down regulation by increased /decreased import or decreased / increased export has to be equal to the difference between the expected wind power production and the actual wind power production. As the model allows wind shedding, the term  $P_{r,s,t}^{WIND,-}$  is added to the equation. If the expected wind power production is higher than the actual wind power production, a demand for up regulation exists. Conversely, there exists a demand for down

regulation if the expected wind power production is lower than the actual one. The heat market is represented in form of an exogenously given demand for each area (4):

$$\sum_{i \in I_a^{HEAT}} Q_{i,s,t} = d_{a,t}^{HEAT} \quad \forall a \in A, \forall s \in S, \forall t \in T \quad (4)$$

The model has the possibility to distinguish between urban and rural areas. The difference is that in rural areas no heat storages are allowed and defined, and that the relative level of the production has to be equal for all units producing heat.

Similar to the heat market, the market for ancillary services is described by two exogenously given demand restrictions for up (5) and down regulation (6). The demand for up regulation, can be supplied either by increased power production of the the power producing units, or by reduced pumping of the hydrostorages, whereas the demand for down regulation can be met by decreasing the power production or by increasing the pumping of the hydro storages..

$$\sum_{i \in I_r^{ELEC}} P_{i,t}^{ANC,+} + \sum_{i \in I_r^{HYDRO}} W_{i,t}^{ANC,+} = d_{r,t}^{ANC,UP} \quad \forall r \in R, \forall t \in T \quad (5)$$

$$\sum_{i \in I_r^{ELEC}} P_{i,t}^{ANC,-} + \sum_{i \in I_r^{HYDRO}} W_{i,t}^{ANC,-} = d_{r,t}^{ANC,DOWN} \quad \forall r \in R, \forall t \in T \quad (6)$$

## B. Capacity restrictions

As the model is defined as a multi-regional model, the capacity restrictions of the transmission lines are defined in (7).

$$P_{r,r,t}^{TRANS} + P_{r,r,s,t}^{TRANS,+} \leq I_{r,\bar{r}}^{TRANSMISSION} \quad \forall r, \bar{r} \in R, \forall t \in T \quad (7)$$

The capacity restrictions for the electricity producing units are defined in the following equations for maximum (8) and minimum electric power output (9).

$$P_{i,t}^{DAY\_AHEAD} + P_{i,s,t}^+ + P_{i,t}^{ANC,+} \leq P_{i,s,t}^{ONLINE} \quad \forall i \in I^{ELEC}, \forall s \in S, \forall t \in T \quad (8)$$

The power, which is committed to the day-ahead market, plus the energy sold at the balancing market for up-regulation plus the contribution to the spinning reserve have to be lower than the capacity currently online of that unit at time step t.  $P_{i,s,t}^{ONLINE}$  is an additional variable introduced in order to describe start-up costs, reduced part-load efficiency and the restrictions for minimum shut down and minimum operation time in a linear programming model. In the typical unit commitment models the restrictions for the minimum operation time and minimum down time include integer variables. However, this is hardly feasible for a model representing a national market. Therefore Weber (Weber, 2004) proposed an approximation to model the restrictions in a linear way, which makes it necessary to introduce this additional decision variable. We want to apply this methodology in this model formulation. On the one hand this capacity online forms an upper bound (8) to the output and on the other hand the capacity multiplied with the quotient of maximum and minimum output forms an lower bound to the possible power output within the model (9).

$$P_{i,t}^{DAY\_AHEAD} - P_{i,s,t}^- - P_{i,t}^{ANC,-} \geq (p_i^{MIN\_PROD} / p_i^{MAX\_PROD}) * P_{i,s,t}^{ONLINE} \quad \forall i \in I_R^{ELEC}, \forall s \in S, \forall t \in T \quad (9)$$

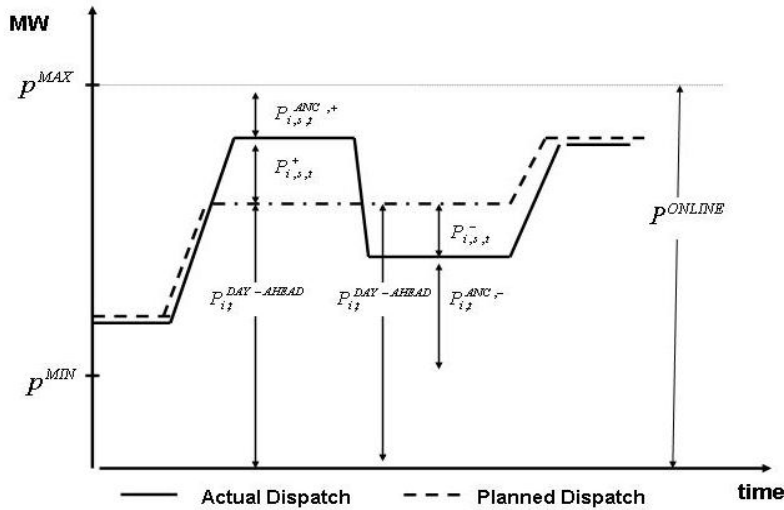
The idea is illustrated in Figure 2.

For CHP units we distinguish between extraction condensing units and backpressure units. The PQ-charts (electric power-thermal power charts) show the possible operation modes of the units in a simplified version representing the possible combinations of electric power and thermal power produced. In Fig. 3 examples of PQ-charts for the two different types of CHP turbines included in the

model are shown. These require additional equations to match these technical restrictions.

For extraction turbines the output of heat and power is restricted by the following three equations (10), (11), (12).

**Figure 2: Illustration of the contribution of a power generating turbine to the different markets**



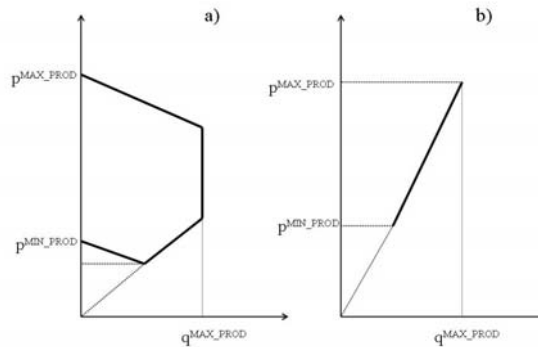
$$P_{i,t}^{DAY\_AHEAD} + P_{i,s,t}^+ + P_{i,t}^{ANC,+} \leq P_{i,s,t}^{ONLINE} - \gamma_i Q_{i,s,t} \quad \forall i \in I^{EXTRACTION}, \forall s \in S, \forall t \in T \quad (10)$$

$$P_{i,t} - P_{i,s,t}^- - P_{i,t}^{ANC,-} \geq \frac{P_i^{MIN\_PROD}}{P_i^{MAX\_PROD}} P_{i,s,t}^{ONLINE} - \gamma_i Q_{i,s,t} \quad \forall i \in I^{EXTRACTION}, \forall s \in S, \forall t \in T \quad (11)$$

$$P_{i,t} - P_{i,s,t}^- - P_{i,t}^{ANC,-} \geq \delta_i^{CB} Q_{i,s,t} \quad \forall i \in I^{EXTRACTION}, \forall s \in S, \forall t \in T \quad (12)$$

$\gamma_i$  corresponds to the electric power reduction due to heat production. For gas-turbines the same restrictions apply with the difference that for gas-turbines  $\gamma_i$  is zero.

**Figure 3: Simplified PQ-chart for a) extraction-condensing turbines and b) back pressure turbines**



Backpressure turbines produce heat and power in relative constant heat ratio  $\delta_i^{CB}$  which is

unit specific. So we get the following equation (13):

$$P_{i,t} + P_{i,s,t}^+ - P_{i,t}^- = \delta_i^{CB} Q_{i,s,t} \quad \forall i \in I^{BACKPRESSURE}, \forall s \in S, \forall t \in T \quad (13)$$

### C. Restrictions for down regulation

For the fluctuating units for which wind shedding is taken into account, the amount of wind shedding has to be lower than the possible wind power production (14).

$$P_{i,s,t}^{WIND,-} \leq P_{r,s,t}^{WIND\_ACTUAL} \quad \forall r \in R, s \in S, \forall t \in T \quad (14)$$

The down regulation for electricity producing units can not be larger than the committed production (15).

$$P_{i,s,t}^- + P_{i,s,t}^{ANC,-} \leq P_{r,t} \quad \forall i \in I^{ELEC}, s \in S, \forall t \in T \quad (15)$$

And also the down regulation by the transmission lines has to be lower than the planned commission (16).

$$P_{r,\bar{r},s,t}^{TRANS,-} \leq P_{r,\bar{r},s,t}^{TRANS} \quad \forall r, \bar{r} \in R, s \in S, \forall t \in T \quad (16)$$

### D. Start-up costs

Start-up costs may influence considerably the unit commitment decisions of plant operators. In unit commitment and load dispatch models, they are typically modelled using binary variables, for unit operation, start-up and shut down. As the size of the power systems require the definition of the model as a linear optimization model, the approximated formulation proposed by Weber (Weber, 2004) for modelling the start-up costs is used in the model. The function of the start-up costs is defined in the following way (17).

$$s_i(P_{i,s,t}^{ONLINE}, P_{i,s,t-1}^{ONLINE}) = \begin{cases} c_i^{START\_UP} * (P_{i,s,t-1}^{ONLINE} - P_{i,s,t}^{ONLINE}) & \text{if } P_{i,s,t-1}^{ONLINE} > P_{i,s,t}^{ONLINE} \\ 0 & \text{else} \end{cases} \quad (17)$$

In order to avoid that units are always kept online, one has to account for that the efficiency at part load is usually lower than at full load. The costs function is defined as fuel consumption function multiplied with the fuel costs. In the case of the approximation the additional decision variable  $P_{i,s,t}^{ONLINE}$  is included into the linear cost function. The cost function for condensing power plants is given in (18).

$$c_i^{OPERATION}(P_{i,s,t}, P_{i,s,t}^{ONLINE}) = e_i P_{i,s,t}^{ONLINE} + f_i P_{i,s,t}, \quad \forall i \in I^{ELEC} \quad (18)$$

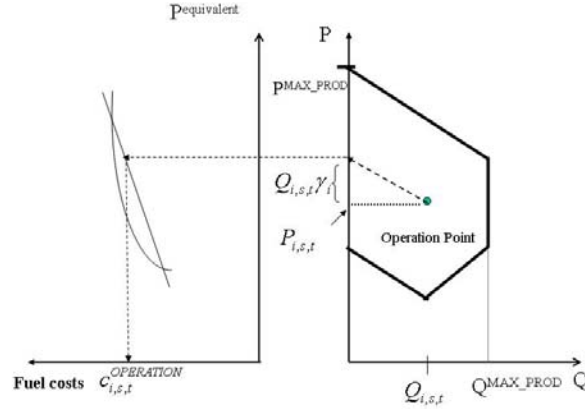
The cost function for condensing-extraction and gas turbines is defined in (19).  $\gamma_i$  stands for the electric power reduction due to heat production. Accordingly the amount of heat production multiplied with the factor  $\gamma_i$  corresponds to the increased fuel consumption caused by the heat production (see Figure 4).

$$c_i^{OPERATION}(P_{i,s,t}, Q_{i,s,t}, P_{i,s,t}^{ONLINE}) = e_i P_{i,s,t}^{ONLINE} + f_i (P_{i,s,t} + \delta_i Q_{i,s,t}) \quad \forall i \in I^{EXTRACTION} \quad (19)$$

As for backpressure turbines the amount of heat output is fixed to power output, we can simplify the cost function for backpressure turbines.

$$c_i^{OPERATION}(P_{i,s,t}, Q_{i,s,t}, P_{i,s,t}^{ONLINE}) = c_i^{OPERATION}(P_{i,s,t}, P_{i,s,t}^{ONLINE}) e_i P_{i,s,t}^{ONLINE} + f_i (P_{i,s,t}) \quad \forall i \in I^{EXTRACTION} \quad (20)$$

**Figure 4: Graphical representation of fuel cost determination for extraction condensing turbine**



#### D. Minimum operation and shut down times

Like start-up costs, minimum operation times and minimum shut down times influence the unit commitment decisions of plant operators. The typical formulation of the minimum operation times restrictions says, that a unit can be shut down, only if it was on during the last  $t_i^{MIN\_OP}$  time steps. In the linear approximation the requirement is, that the reduction in the capacity online of unit I between time step  $t$  and time step  $t-1$  cannot exceed the minimum of the capacity online during the last  $t_i^{MIN\_OP}$  time steps (21). These time steps correspond to the minimum operation hours of the corresponding plan.

$$P_{i,s,t-1}^{ONLINE} - P_{i,s,t}^{ONLINE} \leq P_{i,s,\tau}^{ONLINE} \quad \forall \tau \quad \text{with} \quad t - t_i^{MIN\_OP} \leq \tau \leq t-1. \quad (21)$$

$$\forall i \in I^{ELEC}, s \in S, \forall t \in [t_i^{MIN\_OP}, \dots, T^{OPTIM\_PERIOD}]$$

Conversely the maximum start-up capacity is limited by the minimum of the capacity shut-down during the last  $t_i^{MIN\_SD}$  time steps (22).

$$P_{i,s,t-1}^{ONLINE} - P_{i,s,t}^{ONLINE} \leq c_i^{MAX\_OUTPUT} - P_{i,s,\tau}^{ONLINE} \quad \forall \tau \quad \text{with} \quad t - t_i^{MIN\_SD} \leq \tau \leq t-1 \quad (22)$$

$$\forall i \in I^{ELEC}, s \in S, \forall t \in [t_i^{MIN\_SD}, \dots, T^{OPTIM\_PERIOD}]$$

#### E. Hydro storages

The equations for the hydro reservoirs are summarized in the following four equations. Equation (22) represents the maximum reservoir capacity, the balance equation (23) represents the content of the reservoir capacity taking into account the power production and the energy inflow by the pump process as well as natural water inflow. Equation (24) restricts the capacity of the pumping process and limits the contribution of the pumping process to the down regulating for the balancing market and ancillary services by increasing the pumping process. Conversely the contribution to the up-regulation by decreasing the pumping process is restricted in (25).

$$V_{i,s,t} \leq v_i^{MAX} \quad \forall i \in I^{HYDRO}, s \in S, t \in T \quad (23)$$

$$V_{i,s,t} = V_{i,s,t-1} - (P_{i,t} + P_{i,s,t}^+ - P_{i,s,t}^-)(1 - \varepsilon_i) + (W_{i,s,t} - W_{i,s,t}^+ + W_{i,s,t}^-) + i_{s,t}^{INFLOW} \quad (24)$$

$$\forall i \in I^{HYDRO}, s \in S, t \in T$$

$$W_{i,t} + W_{i,s,t}^- + W_{i,t}^{ANC,-} \leq w_i^{MAX} \quad \forall i \in I^{HYDRO}, s \in S, t \in T \quad (25)$$

$$0 \leq W_{i,t} - W_{i,s,t}^+ - W_{i,t}^{ANC,+} \quad \forall i \in I^{HYDRO}, s \in S, t \in T \quad (26)$$

## F. Data aggregation

To reduce the computation time of the model, several district heat areas are summarized. For areas defined as rural areas, it is required that the heat output is proportional to all units. This is done in order to avoid that only the cheapest CHP units are running. The equations about data aggregation are omitted here and can be found in the model code for the Balmorel model (Balmorel, 2001).

## G. Non-anticipativity constraints

As we use a multi-stage scenario tree to model uncertainty instead of single scenarios, we have to enforce that the decisions taken at time  $t$  must be the same if two scenarios are indistinguishable until time  $t$ . These sets of restrictions are known as non-anticipativity constraints. The detailed formulation of the restrictions can be found e.g. in Takriti et. al (2002).

# 3 Scenario creation and scenario reduction

The inclusion of the uncertainty about the wind power production in the optimization model is considered by using a scenario tree. The construction of this scenario tree is carried out in two steps:

- A. Modelling of the wind power generation data process and the simulation of the independently, identically distributed scenarios, whose first values at the root node are identical
- B. Reduction of the scenario tree

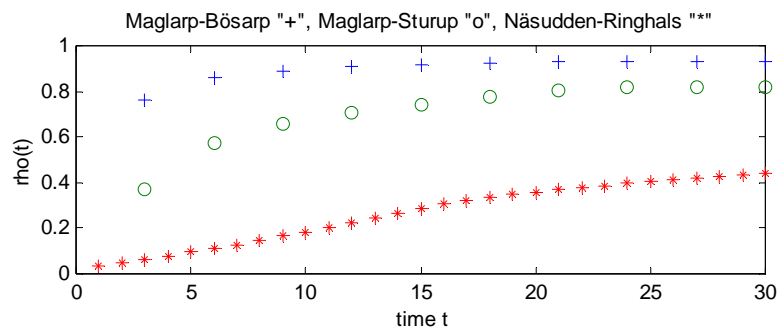
In the following these steps are described in detail:

### A. Modelling the wind power generation data process

The wind power generation model is based on data about wind speed which can be obtained in two different ways: either directly from a database of meteorological data or as output of the Power-to-Speed Model (Norgard et. al., 2004). The latter reconstructs the wind speed from given regional wind power data. Depending on the region, one of these two alternatives is possible. So in each case the wind speed data series are obtained on the basis of historical information.

A second important source of data is a database about the historical forecast errors for the wind speed. The errors between the wind speed forecasts and the real wind speed can be quite large. So it is of crucial importance to include prediction errors in the model. Since the errors increase with the length of the forecast period, the so called “Wind Speed Forecast Error Module” (Söder, 2004) assumes a multidimensional ARMA time series for this forecast error for each station additionally taking into account the correlation between different stations. For example, data analysis from Sweden (see Fig. 5) shows a) that the closer the stations, the higher is the correlation between forecast errors and b) that the correlation between different station increases with forecast lengths.

**Figure 5: Correlation between forecast errors for different pairs of stations**

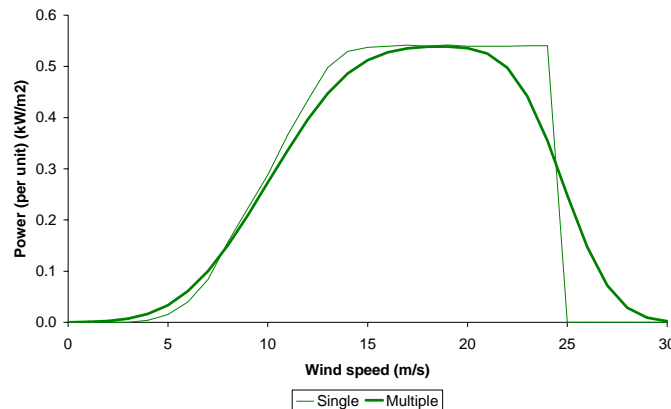


Source: Söder (2004)

The ARMA time series contain the usual error terms. These are simulated by Monte Carlo Simulations resulting in a large number of scenarios for the forecast error.

In order to obtain, for each region, the forecast for the wind power from the wind speed forecast, technological aspects of the wind power stations located in the considered region are needed. Additionally, their spatial distribution within each region has to be taken into account. This yields an aggregation of the power generation in each region by smoothing the power curves (see Fig. 6).

**Figure 6: A standard normalised power curve ('Single') and the corresponding smoothed power curve ('Multiple').**



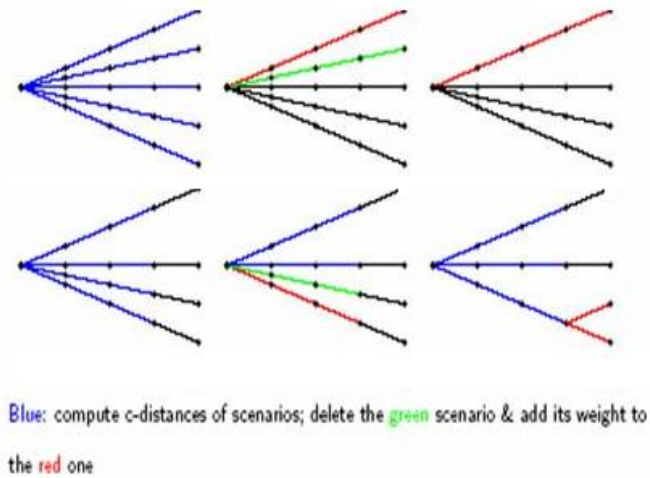
Source: Norgard et al. (2004)

**B. Scenario reduction:**

In order to keep computation times small for models representing a national market with a huge number of generating units, only significantly less scenarios than the scenarios created by Monte Carlo simulations before can be used. Therefore we use a stepwise backward scenario reduction algorithm based on the approach of Dupacova et al. (2003): the original scenario tree is modified through bundling scenarios or part of scenarios.

As a measure for the similarity of different scenarios, the Kantorovich distance between two scenarios is used. The reduction algorithm proceeds backwards: in the first iteration, the calculation of the distance between two scenarios includes all stages, in the second iteration, all stages except the last one are considered, etc. Merging two scenarios or parts of scenarios means deleting the one (or the part of the scenario) with the lower probability and adding its probabilities to the remaining one (see Fig. 7). Note that merging scenarios in the second or any following iteration changes the successors of nodes in the scenario tree.

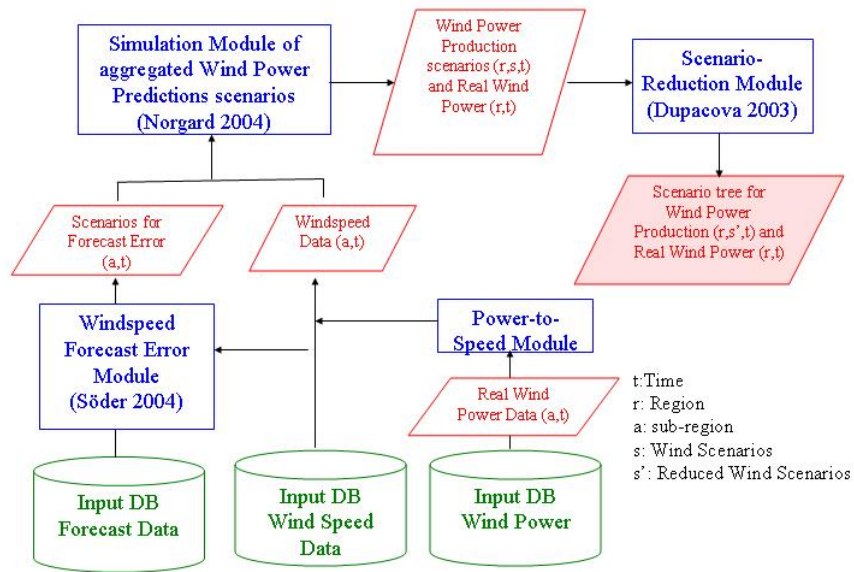
**Figure 7 : Example for the backward scenario reduction heuristic**



Source: modified figure from Gröwe-Kuska et al. (2001)

An overview of the different modules and the data flow can be found in Figure 8. All modules are implemented within Matlab®.

**Figure 8: Data flow of the sub-module of the scenario tree creation model**



## 4 Application

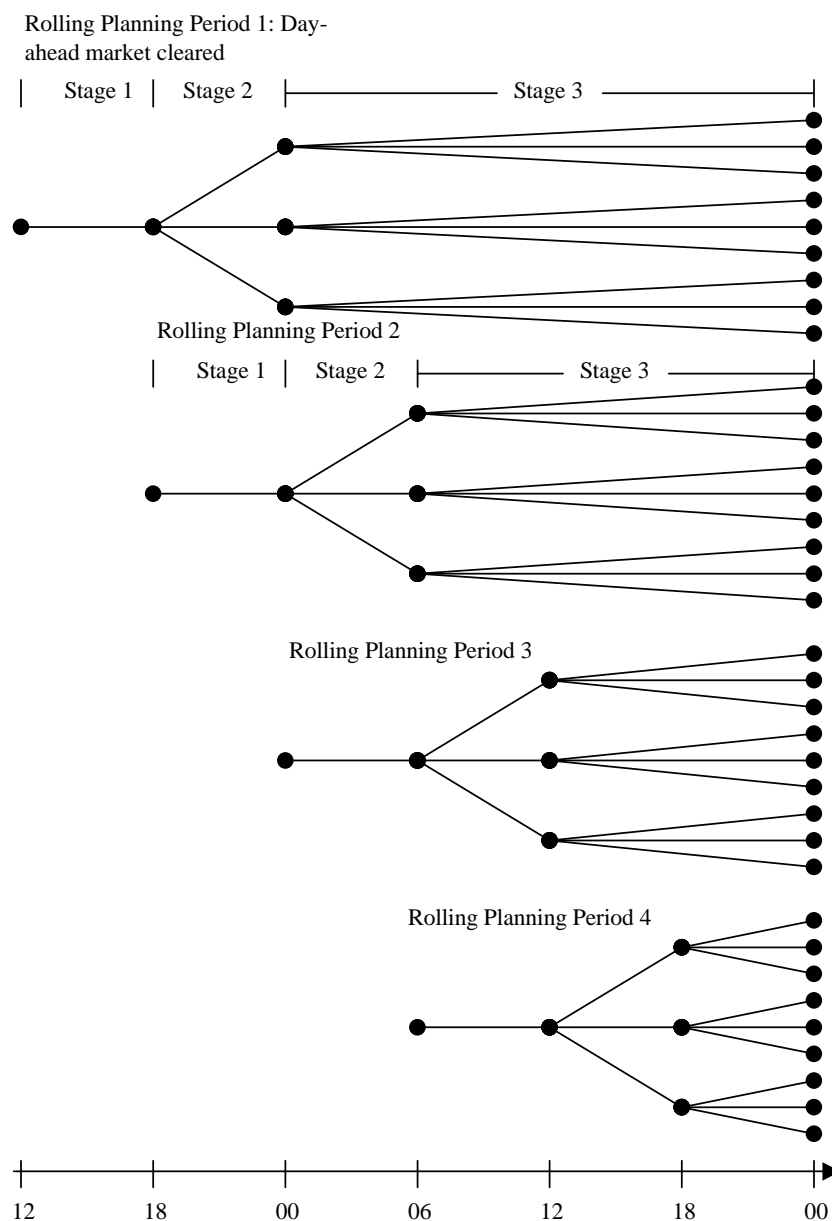
The methodology proposed above has been applied to the German market. The market for balancing power is assumed to function as an hour-ahead market in Germany – although such a market is - in contrast to the Nordic countries – is not yet established in Germany.

Three regions were defined according to the bottlenecks of the transmission lines between the coastal areas with huge installed wind generation capacities and the main consumption areas in the midland. Each region is divided into an urban and a rural area. 40 different types of units were considered. For representing the impact of the uncertainty of wind power production on the trading activities

on the different markets, at first an analysis of the decision structure has to be performed to clarify the decision structure of the problem.

In general, new information arrives on a continuous basis and provides updated information about wind power production and forecasts, the operational status of other production and storage units, the operational status of the transmission and distribution grid, heat and electricity demand as well as forecasts and updated information about day-ahead market and regulating power market prices. Thus, an hourly basis for updating information would be most adequate. However, stochastic optimisation models quickly become intractable, since the total number of scenarios has a double exponential dependency in the sense that a model with  $k+1$  stages,  $m$  stochastic parameters, and  $n$  scenarios for each parameter (at each stage) leads to a scenario tree with a total of  $s = n^{m^k}$  scenarios. It is therefore necessary to simplify the information arrival and decision structure in the stochastic model (cf. Fig. 9).

**Figure 9: Illustration of the rolling planning and the decision structure in each planning period within a day.**



In the current version a three stage model is implemented. The model steps forward in time using rolling planning with a 6 hour step. This decision structure is illustrated in **Fehler! Verweisquelle konnte nicht gefunden werden.** showing the scenario tree for four planning periods covering

one day. For each planning period a three-stage, stochastic optimisation problem is solved having a deterministic first stage covering 6 hours, a stochastic second stage with three scenarios covering 6 hours, and a stochastic third stage with 9 scenarios covering a variable number of hours according to the rolling planning period in question. In the planning period 1 the amount of power sold or bought from the day-ahead market is determined. In the subsequent replanning periods the variables standing for the amounts of power sold or bought on the day-ahead market are fixed to the values found in planning period 1, such that the obligations on the day-ahead market is taking into account when the optimisation of the intra-day trading takes place.

For the analysis, Germany is divided into three regions: one for the coastal areas in the north-west and the north-east each and a third, larger one for the central and southern part. This reflects the concentration of the wind power production in the coastal areas and the existing bottlenecks in the German power transmission system.

First results for the day-ahead market prices on a winter day are given in Figure 10. Obviously, the transmission restrictions between the regions are binding, since the power prices between the regions deviate substantially. As expected for strong wind situations, the power prices are usually lower in the windy regions than in the central-south region with less wind power production. Yet some price-spikes occur in the north-west region, which correspond to start-up of conventional units to meet peak demands.

**Figure 10: Day-ahead market price derived from the model in the three regions (Central-South, North-East, North-West) of Germany**

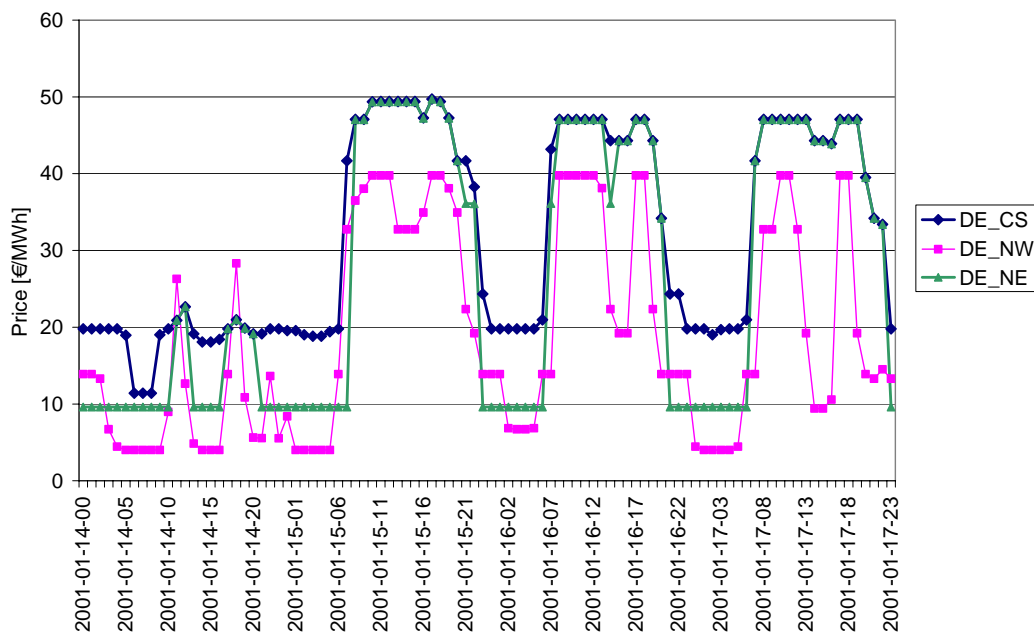
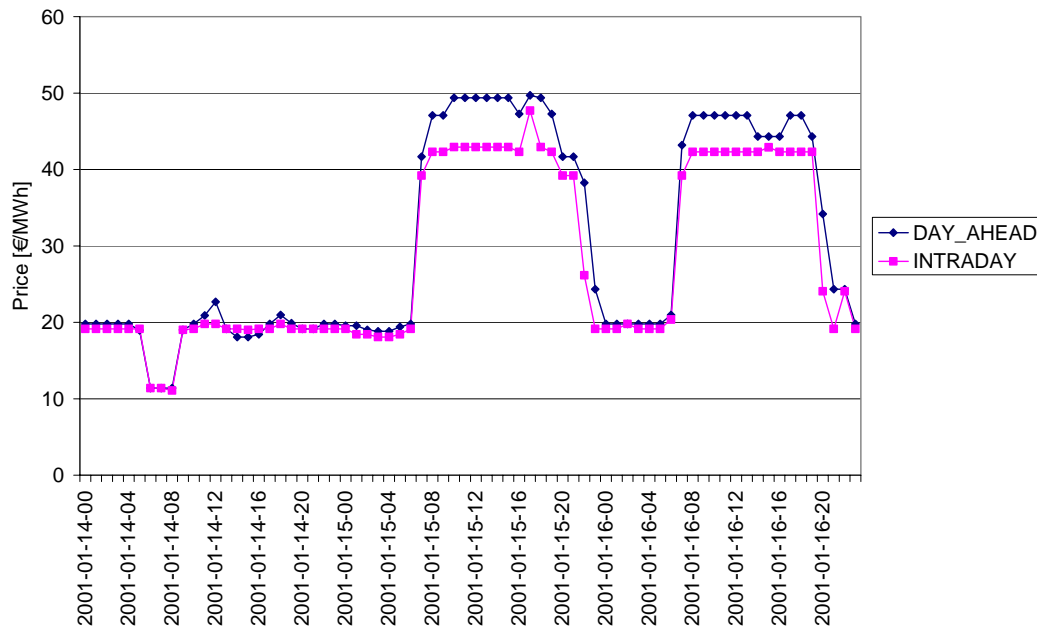


Figure 11 illustrates the price variation between the day-ahead and the intraday market. The prices are found to be mostly similar but not the same. If the actual wind power production is larger than expected, the intraday market price is smaller than the day-ahead market price since the latter is based on the prediction of smaller values for the wind power production. In such situations a need for down regulation exists. If on the other hand, the real wind power production is smaller than expected, it is the day-ahead market price that is smaller. The intraday market price is based on additional information about the wind power production than the day-ahead market price. Indeed, it is the error in the forecast of the wind speed that causes the error in the prediction for the wind power production. So this justifies the detailed treatment of the error in the wind speed forecast in this model.

For the detailed analysis of the impact of wind integration on system operation and system costs further analysis for other typical days are obviously required, which we expect to carry out in the near future.

**Figure 11: Day-ahead market prices and intraday market prices as marginal costs for Central-South of Germany**



## 5 Final remarks

In this paper a stochastic linear programming model for evaluating the impact of wind power integration has been developed. The model describes efficient, optimized decision making in the power markets using rolling planning. The model considers explicitly the interplay between four markets: i) a day-ahead market for physical delivery of electricity, ii) an intraday-market, iii) a day-ahead market for automatically activated reserve power and iv) a market for district heating and process heat. The inclusion of CHP allows to analyse the potential usefulness of CHP production with heat storages for balancing fluctuating wind energy. By using detailed time series for the wind speed data and modelling the forecast error of wind speed, the impact of wind uncertainty on system operation, prices and integration cost can be evaluated in detail. In order to keep computing times small, scenario reduction algorithms are applied.

So far only a first application of the model to the German market has been done, but with future applications detailed and methodologically sound estimates on the impact of wind power integration on system operation and costs will be derived, contributing hence to identifying efficient strategies for integration of renewables.

## Acknowledgements

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